SIMULATION OF FATIGUE CRACK PROPAGATION UNDER CONTACT LOADING CONDITIONS

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Abstract Numerical simulations of pitting arise phenomena on the gear teeth is presented in the article. The presented approach is based on the assumption that pitting (pits) are a result of fatigue crack propagation under complex (rolling, sliding, lubricated) contact loading conditions. The solution approach consisted of numerical simulations of fatigue cracks growth in the FEM framework. Penetration of the fluid lubricant into fatigue (pitting) cracks is simulated using special cavity finite elements, which allow to include so called lubricant closure inside crack. Fatigue (pitting) crack growth predictions are based on the Paris law in conjunction with FEA of fracture criteria. The simulations were carried out under the commercial code ABAQUS CAE FEM programme which allows to develop in-house codes using the Python scripting language and which is the basis of all FEA including the simulation of the contact loading conditions and the crack growth. An acceptable agreement of the numerical and experimental results confirms (not proves) that fatigue crack propagation can present a true damage mechanism of the pitting wear rise.

1 INTRODUCTION

Pitting is understood as fatigue damage of components caused by cyclic contact load, when material particles come off and shallow pits arise in contact surface. Pitting is most often related to damage of contact surface of gear teeth, but it also occurs on working surface of rolling bearing or on heads of rail and railway's wheels. Pitting formation leads to degradation (in extreme case to loss) of functionality of afflicted device, to escalation of vibrations, noise and other negative effects. Therefore, appreciable attention is given to study of this damage with purpose of reducing or eliminating these negative effects.

Fundamentals of pitting formation are so complicated, that there are still being used special empirical relations for determination of lifetime (of for example gear sets), which require experimental data gained from tests carried out directly on gear sets in special test stands. These experiments are time-consuming and also costly.

There have been appearing efforts at mathematical description of pitting wear in recent years. These attempts result from hypothesis that initiation and fatigue crack propagation are in progress before pit rise. The experiments indicate that under certain conditions crack propagates from contact surface into depth of material at first then it is curved and turned backward towards the surface. Finally, end-fracture results in detachment of a material particle, as a result of which the pit is originated.

The published approaches to the numerical simulation of pitting damage rise process can be divided into two main domains. The basic assumption of the first group is that a pressured fluid lubricant penetrates into cracks and influences its growth. This approach can be represented e.g. by Fajdiga et al. (2004), who simulate fatigue crack growth from a surface initial crack and contact loading approximated by pressure distribution corresponding to the EHD lubrication theory. In the second group of approaches other possible damage mechanisms of pitting rise are assumed. E.g. Ding et al. (2003) simulate pitting rise from subsurface initial cracks.

The presented article belongs to the first group of the above mentioned approaches. The fatigue crack growth is numerically simulated by Finite Element Method. The crack growth starts from the surface initial crack. The contact conditions between real gear teeth near the pitting crack mouth were computed and the pressured fluid lubricant penetration into pitting crack was assumed.

2 IMPLEMENTED PHENOMENOLOGICAL CRACK GROWTH THEORIES

Validity of the small scale yielding conditions is the basic assumption of the presented crack growth simulations. The phenomenological theory of fatigue crack propagation – the Paris law – applied in this work supposes that the initial cracks are so long that all parts could be modelled as an isotropic continuum and a dimension of crack tip plastic zone is negligible compared to the crack length.

Standard Paris law equation is written as relation of crack growth rate again stress intensity factor amplitude. In the presented article the crack growth predictions were based on the computation and evaluation of fracture mechanics J-integral criterion. The application of J-integral criterion enforces the modification of Paris law relation, which consist in substitution of stress intensity factor amplitude by amplitude of J-integral (1).

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C'(\Delta K)^{m'} \Rightarrow \frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta J)^{m} \tag{1}$$

The reason of mentioned modification results from application of specific loading conditions in the computational model (gear meshing simulation), which has to be solved under large deformation condition (nonlinear solution) and therefore computation of stress intensity factor is not support.
Thus the J-integral was used for both calculating fatigue crack growth rate according to the Paris law (1), where C and m are material parameters and ΔJ is a J-integral amplitude, and evaluating the fatigue crack growth direction, which corresponds to the direction in which the maximum of the J-integral value was calculated, Figure 1. This criterion is equivalent to the maximal tangential stress criterion in terms of linear elastic fracture mechanics conditions.

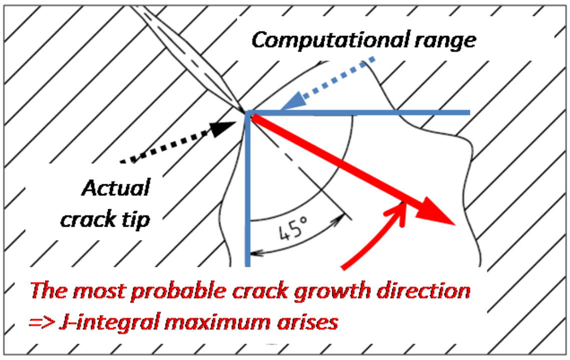

Figure 1. Evaluation of crack growth direction.

3 EXPERIMENTAL WORKS

Experimental works were carried out at two basic levels. Firstly, simple so called CT test specimens were employed to provide data for validation of the crack growth prediction models. Secondly, a setup of gearing testing machine was debugged and fatigue tests of the real gearing were carried out.

3.1 CT specimens testing

The main goal of the CT specimens testing was both the Paris law parameters identification used for the crack growth rate prediction and verification of the implemented approach to crack growth direction prediction (maximum of J-integral) according to observing the crack growth under mixed mode loading conditions (Španiel et al. 2008). The CT specimens (Figure 2) were manufactured from the 18CrNiMo7-6 material, which is normally used for gears and shafts. After heat finishing, the following characteristics of the material can be mentioned: Young's modulus 210 000 MPa, yield stress 1100 MPa, strength 1250 MPa. The estimated Paris law parameters are the following: C=1.42e-5 mm/[cycle.(N.mm)$^{1.3}$] and m=1.3.

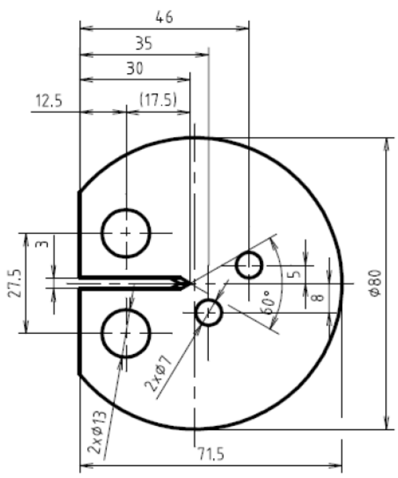

Figure 2. CT specimen for observation of crack growth direction evolution under mixed mode loading conditions.

3.2 Real gearing testing

The main goal of the gearing testing was to obtain experimental data for subsequent verification of the numerical model of the fatigue crack growth under contact loading conditions. Verification criteria of numerical models were both the geometrical parameters of pits and the assessment of the number of the loading cycles, which are necessary for the pits rise. Parameters of tested gearing are listed in Table I.

Table I. Geometrical parameters of tested gearing.

Gearing ratio	i_{mp}	3,2
Modulus	m	5 mm
Axis distance	a_w	160 mm
	Pinion	Gear
Teeth number	15	48
Gear thickness	17 mm	12 mm

The whole gearing test equipment is shown in the Figure 3. It was designed as so called Niemann closed testing chain, which consists of two gearboxes - technological and testing one, Petr, K. et al. (2010). Gearboxes are connected with two shafts, which create closed torque loading loop. The required test toque is induced by a leverage mechanism.

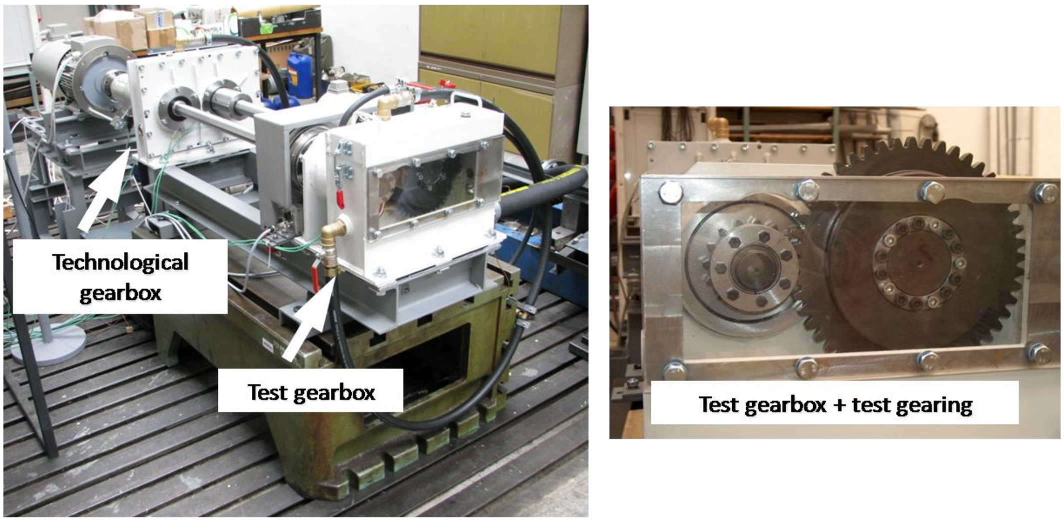

Figure 3. Gearing testing equipment.

Basic technological treatment of gearing was performed. The material of gear teeth cores were tempered and contact surfaces were cemented into deep approximately 0.7 - 0.9 mm and hardened to final hardness approximately 58-60 HRC. Six tests were carried out. Digital pictures of teeth contact surfaces were made in the fixed time interval during the testing with time increment 2.5 hours. An example of time series of surface fatigue damage pictures is shown in the Figure 4. Tested gearings were designed according to norm ISO 6336/1996 to limit durability approximately 30 hours under given loading conditions. Each test was stopped after formation of large pit or pits (maximal permissible pitting damage pollution of contact surface is 4%). The average life time of tested gearing was approximately 54 hours. The higher durability could be explained by the conservative design approach based on the ISO norm.

For more details some pits geometry and pitting cracks in various propagation stages were analyzed using suitable material cuts and metallographic samples of the most damaged gear teeth as is shown in the Figure 5. According to detailed pictures shown in the Figure 5 can be condlude:

- Pitting cracks start to propagate especially from the so called one tooth contact condition area, thus the pitting damage is concentrated in this domain.
- Relatively high smooth of fracture surface could be explain by contact loading during crack propagation (detail A, Figure 5).
- Significant crack tip striations can identified inside pit (detail B, Figure 5). This suggests, that the pit formation is a result of crack propagation in the direction from teeth heel to head (detail C, Figure 5).
- On the basis of relatively straight crack mouth could be assumed, that before magistral crack propagation many short crack exist in the critical domain of contact surface, which can subsequently create magistral crack.

Further it can be derived from the metallographic samples that the fatigue cracks are initiated along the whole contact surface. In the tooth heel domain the cracks are initiated in the direction of the tooth head and in the tooth head domain in the direction of the tooth heel. The results of the experimental tests show

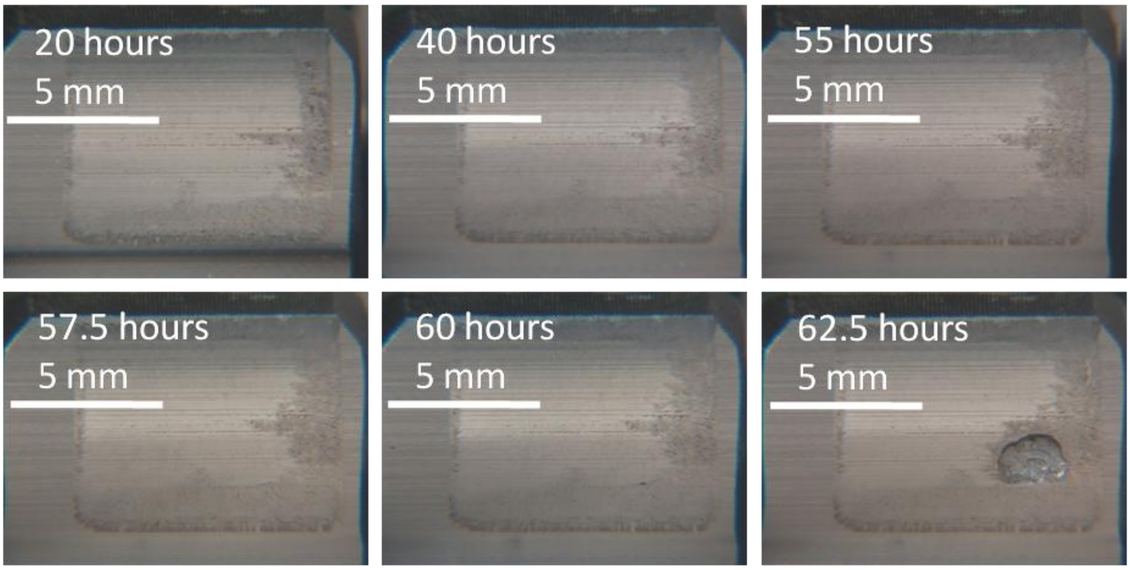

Figure 4. Fatigue damage of contact surface depending on time.

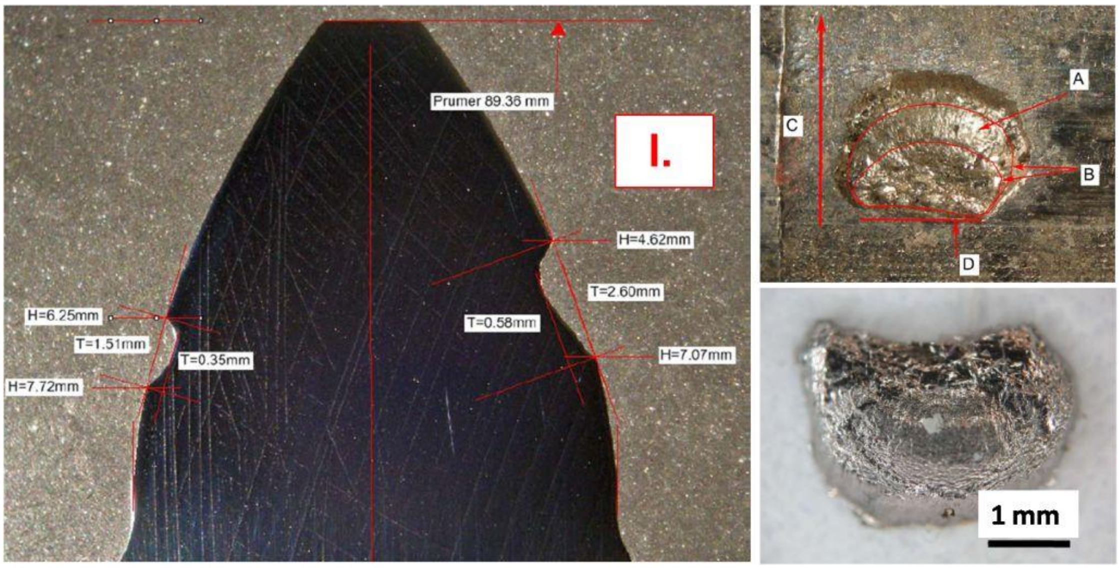

Figure 5. Detailed pit evaluation.

4 NUMERICAL SIMULATIONS

All simulation were performed under the commercial FEM code ABAQUS, which provides both computation tools for evaluation fracture mechanics criteria (stress intensity factors *K*, T-stress, J-integral) and Python language interface for in-house programme codes submitting.

The investigation of pitting formation presented in this article is based on the simulations of the gearing contact conditions, which induce boundary conditions for subsequent fatigue crack growth computational predictions. Basic mechanical quantities defining contact conditions are: contact pressure, shear stress, relative slip range and rate. Actually these quantities could be affected by both properties of fluid lubricant used and contact surface roughness. In the FEA models the complex tribological relations (so called EHD lubrication conditions) are approximated by friction coefficient *f*, whose value can be in the range of 0.02-0.1.
Linear elastic isotropic material model was assumed and all FE analyses are assumed as quasistatic and planar considering plane strain conditions.

According to both published works and own experiences the penetration of pressured fluid lubricant into pitting cracks should be considered in simulations, because the pressured lubricant can cause crack opening during contact rolling around the cracked region. A cavity model was introduced to include pressured fluid lubricant penetration into the pitting cracks during simulations. The pressure of the lubricant penetrated inside the crack is assumed to be equal to the actual contact pressure near the crack mouth. The change of contact pressure distribution under EHD lubrication conditions was not assumed.

A complex in-house programme code was developed at the Faculty of Mechanical Engineering CTU in Prague for executing pitting fatigue crack growth simulations under software ABAQUS. This script code provides full automatic control of whole simulation. The brief block scheme of simulation run is shown in the Figure 6.

At first the parametric FEM model of given gearing is created and simulation of contact conditions around the future crack mouth location is performed to estimate the relevant contact pressure distribution (steps 1.1 and 1.2 in the Figure 6). During this simulation pitting crack is not included in the model.

Subsequently the simulations of pitting crack growth are performed. The initial crack is included during the initial computational step. According to the experimental observation the initial crack was placed in the contact surface of the pinion. Its position corresponds with a line labeled as D in the Figure 5 and its inclination to the contact surface is 45° in toward the tooth head (Figure 7).

In the subsequent computational steps the crack is incrementally extended by a relatively small length increment equal to 0.015 mm in the given direction. Each computational step consists of:

- the contact rolling simulation around the crack region (step 2.1, Figure 6). The whole FEM model is shown in the Figure 8.
- J-integral amplitude ΔJ and crack growth direction evaluation and calculation of loading cycles number needed to the given crack extension (step 2.2, Figure 6)
- the pitting crack geometry modification (step 2.3, Figure 6).

Influence of the pressured fluid lubricant closed inside the crack is included in the FE models using special hydrostatic fluid elements (F2D2 in ABAQUS), which allow simulating charging resp. discharging of deformable cavities by uncompressible fluid with consideration of viscous resistance. Implementation of mentioned cavity model results from the basic assumption, that the crack can growth only if it is opened. Thus if the contact between crack surfaces arise (pressure in the cavity is not able to open crack), then the crack can not growth.

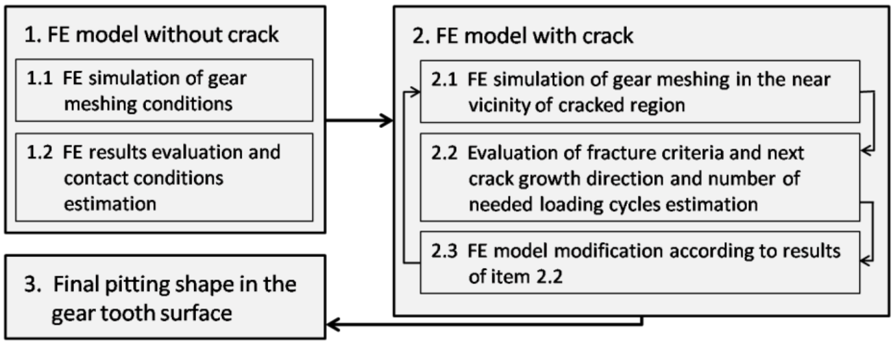

Figure 6. Block schema of pitting simulation under ABAQUS.

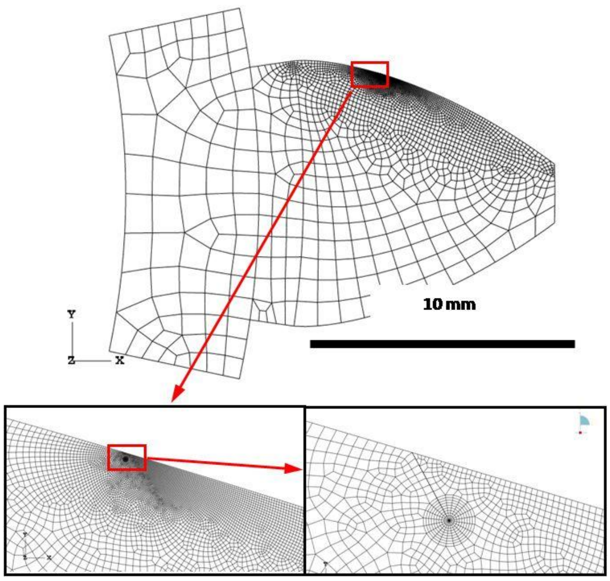

Figure 7. Block Initial crack position in the pinion contact surface.

With respect to the complex physical background leading to pitting formation, which can not be fully experimentally explained, the developed FEM model was created in the relatively general form. This means, that they contains many parameters, which can be change independently and its influence on the pitting crack growth can be evaluated. The basic model parameters are listed in the Table II.

Table II. Basic FEM model parameters.

FEM model parameter	Min value	Max value
Friction coefficient on the teeth contact surfaces [-]	0.01	0.15
Friction coefficient between crack surfaces [-]	0.3	1
Initial crack length [μm]	10	50
Initial crack angle between crack and contact surface [°]	25	90
Crack extension increment length [μm]	10	50
Initial crack orientation	To the head	To the heel
Initial crack location on the contact surface (distance from the pinion center) [mm]	36	44
Pressured fluid inside the crack	NO	YES
Single cavity cell OR multiple connected cavity cells (special cavity cell is created during each crack extension) inside the crack		
Viscous resistance coefficient []	≈ 0	10^6

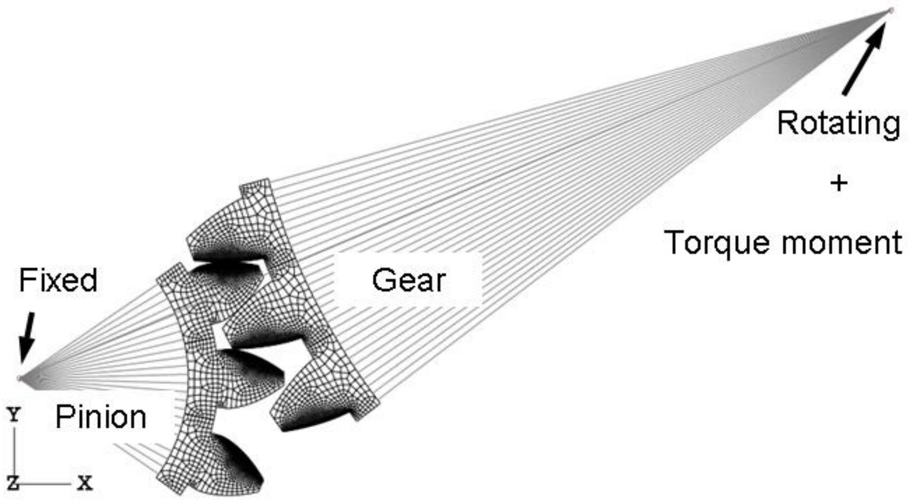

Figure 8. Whole FEM model of gearing.

5 RESULTS AND CONCLUSION

Investigation of pitting damage phenomenon is based on both experimental results of real gearing tests and numerical simulations of crack propagation under contact loading conditions.

From picture series (an example is show in the Figure 4) results, that before a relatively large pit appears on the contact surface, several small (microscopic) initial pits rises around initial crack mouth (line D in the Figure 5). After formation of small pits, the large pits are created in relatively short time. In addition, according to material particles, which were found on the bottom of gearbox and metallographic samples (Figure 5), be conclude following:

- At first initial small cracks forms around whole contact surface.
- These initial cracks are leading to the formation of small initial pits in the most critical regions. Very small material particles are removed from the contact surface.
- Then main crack can propagate into material from the bottom of initial small pits.
- After the main crack reaches a critical length, relatively large material particle breaks off from the contact surface and pit appears.

The main goal of performed numerical simulations is to provide information, which can confirm or confute above mentioned theory of pitting damage formation. The simulated crack behavior (crack growth direction) is analyzed with respect to:

- the location of contact region between gear teeth according to the crack mouth,

- gearing meshing conditions,
- the stress field around the whole crack,
- the possibility of fluid lubrication penetration inside the crack and
- the J-integral value.

In the first step simplified FEM model of pitting cracks was applied. Formation of initial small pits was neglected. Penetration of pressured fluid lubricant into crack was simulated by using hydrostatic fluid elements (F2D2 in ABAQUS), which created simple cavity volume inside the crack. The pressure of lubrication inside the crack is considered to be equal to actual contact pressure in the crack mouth. The charging resp. discharging of crack cavity volume is determined by pressure drop between inside crack volume and pressure in the crack mouth.
The FEA model of pinion with the final crack (predicted pit shape) after the crack growth simulation is shown in the Figure 9. Estimation of the number of loading cycles was based on the J-integral amplitude calculation in each simulation step. The crack length increment Δa was every time equal to 0.015mm. The predicted number of loading cycles required for the pit rise according to the Figure 9 is approximately equal to 2.5E6, which corresponds to the 28.6 hours of experimental testing. The summation of crack growth simulation results is listed in the Table III. The computed J-integral value were used to calculation of number of loading cycles, resp. hours of experimental testing.

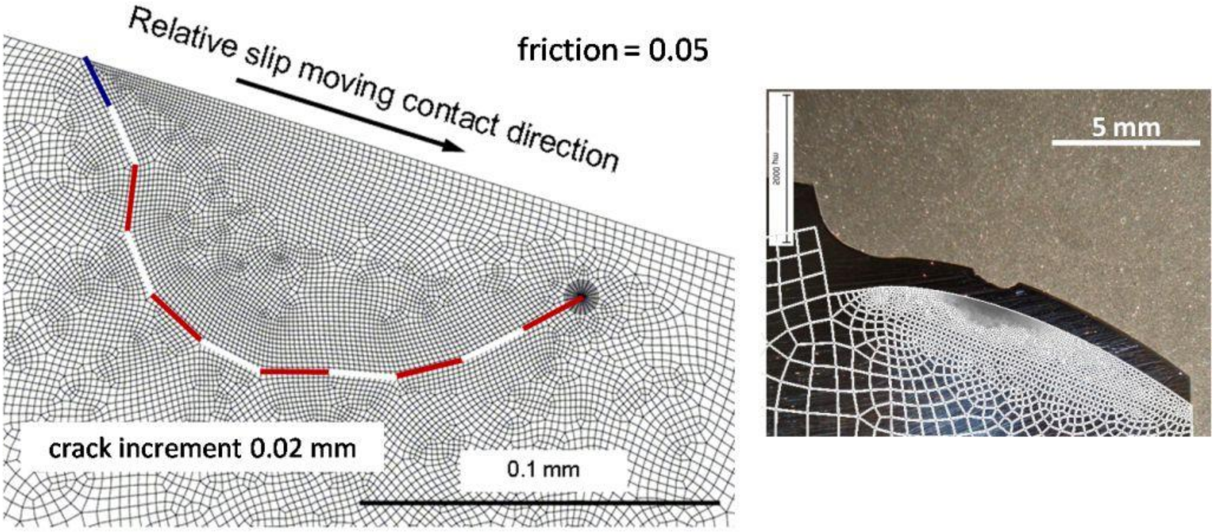

Figure 9. Simulation pitting crack formation.

Table III. Summation of crack growth simulation.

Crack length increment	Crack length [mm]	J-integral value [Nmm]	Crack rate [mm/cycle]	Number of loading cycles [-]	Time [hours]
Initiation	0.015	-	-	-	-
1	0.035	0.004179	1.14E-08	1.75E+06	20.08
2	0.055	0.026429	1.26E-07	1.59E+05	1.83
3	0.075	0.019907	8.71E-08	2.30E+05	2.64
4	0.095	0.042368	2.33E-07	8.60E+04	0.99
5	0.115	0.044134	2.45E-07	8.16E+04	0.94
6	0.135	0.08093	5.39E-07	3.71E+04	0.43
7	0.155	0.082998	5.57E-07	3.59E+04	0.41
8	0.175	0.105771	7.64E-07	2.62E+04	0.3
9	0.195	0.085879	5.83E-07	3.43E+04	0.39
10	0.215	0.064731	4.03E-07	4.96E+04	0.57
Total				2.49E+06	28.58

According to the achieved results can be conclude, that the dimensions of the final pit (Figure 9) is approximately 10x smaller and the predicted time for pitting rise is approximately 3x shorter than the experimental results.

These differences could be caused by a simplified FEA model. In details can be mentioned following simplifications:

- Hydrodynamics effects between gears teeth are approximated by dry friction with appropriate friction coefficient.
- Heat treatment of contact surfaces has not been included yet.
- Simplified cavity model is implemented, which is not able to simulate gradual pressure penetration inside the crack with respect to variable pressure drop along crack length.

The main result of the above mentioned simulation is confirmation that the Paris law and fracture mechanics approach can be a relevant mathematical model to describe the pitting crack growth under rolling contact conditions.

The heat treatment of contact surfaces and more sophisticated cavity model will be implemented in the near future to refine achieved results and expand theoretical knowledge about pitting damage formation.

6 ACKNOWLEDGEMENT

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